# A Many-to-Many Vehicle Routing Problem with Split Loads 

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Keywords: split pickups and delivery, vehicle routing, heuristic, tabu search, bike-sharing

## Introduction

In the classical Vehicle Routing Problem (VRP), each customer is required to be visited by exactly one vehicle and the objective is to minimize the total distance traveled. This implies that (1) no splitting of loads is allowed; and (2) it is a one-to-one shipment. Though, in the real world, truck operators allow the excess capacity in the truck to be filled using partial loads in order to increase efficiency and serve many-to-many shipments. Several (one-to-many) studies have found that the VRP with split deliveries reduces the routing cost compared to the case where a single visit to each customer is imposed in the traditional VRP (Archetti et al., 2008).

The Bike-sharing Rebalancing Problem (BRP) represents a Many-to-Many Split Pickup-andDelivery Problem with One Commodity (M-MSPD-OC), where both the pickup and the delivery demand are allowed to be split, and the pickup-delivery pairs are many-to-many. Self-service parcel lockers were launched by the Dutch postal operator (PostNL) at several public locations in Amsterdam to allow customers to pick up parcel and commerce purchases as well as send parcels, 24 hours a day (PostNL.com, 2019). We call the parcel station problem a Many-to-Many Split Pickup-and-Delivery Problem with Fixed Pairwise Demand (M-MSPD-FPD).

In this study, we first define and formulate a general Many-to-Many Split Pickup and Delivery Problem (M-MSPDP). Because the problem is NP-hard (Dror et al., 1994), we propose a heuristic called Maximum Split-Benefit with Tabu Search (MS-BTS) to efficiently solve for a large-scale M-MSPDP-FPD, which can be applied iteratively to solve for M-MSPD-OC. This study contributes to the vehicle routing literature by introducing a heuristic for solving the general case of M-MSPDP.

## Methodology

In general, M-MSPDP is represented with a directed complete graph, $\mathbf{G}=\{\mathbf{N}, \mathbf{A}\}$, where $\mathbf{N}$ is a set of vertices, $\mathbf{N}=\mathbf{C}+\{0\}$, and $\mathbf{A}$ is a set of edges, $\mathbf{A}=\left\{a_{i j}=\left(c_{i}, c_{j}\right), \forall c_{i}, c_{j} \in \mathbf{C}, i \neq j\right\}$. For each edge $a_{i j}$, there is a cost associated with it. This cost can be measured in terms of distance or travel time between $c_{i}$ and $c_{j}$, or labor cost (driver's wage, $C_{T}$ ), or some generalized cost. Thus, solving the M-MSPDP finds a strategy of truck dispatching and routing and load splitting in order to minimize total cost incurred by truck routing. the working hours for the truck are restricted to 8 hrs. We propose the Maximum Split-Benefit with Tabu Search (MS-BTS) heuristic to solve the M-MSPDP. MS-BTS is built on the Pickup and Delivery Problem with Split Loads (PDPSL) heuristic presented in Nowak et al. (2008).

[^0]The PDPSL heuristic is presented by Nowak et al. (2008) to solve large scale pickup and delivery problems with split loads. Using hypothetical problem sets, Error! Reference source not found. presents the results of the numerical experiment based on the PDPSL heuristic. It shows the average percentage increase in cost when split loads are not allowed for the 75,100 and 125 request problem set ( 5 origins and 15, 20 and 25 destinations) with different load size ranges (in terms of Truck Load, TL) for an average of 30 instances for each load range of every problem set.


Figure 1: Average percentage cost increase without split loads relative to the cost with split loads in three $O-D$ matrices for 75,100 and 125 pairs

The general idea of MS-BTS heuristic is as follows.
Step 1: First, an initial solution is generated by creating dedicated routes for each pickup-delivery pair in the problem.

Step 2: Then these routes are split and consolidated by performing the following. First, based on Figure 1, a random load is selected from the range of $0.51-0.6 \mathrm{TL}$ to be considered for split, based on the additional cost of generating the split. The load and the associated cost are recorded in the tabu list to prevent the same load being selected repeatedly.

Step 3: The routes are then combined based on the reduction in cost and subject to vehicle capacity along the route, using the Clarke and Wright's savings algorithm.

Step 4: Local search procedures are applied in the following order - intra-route load swaps, interroute load swaps, intra-route load insertions, inter-route load insertions, reordering of origins and destinations.

Step 5: Another load not present in the tabu list is selected and the process is repeated from Step 2.

Once the loads in the range of $0.51-0.6 \mathrm{TL}$ are exhausted, loads are selected from the range of $0.61-0.7 \mathrm{TL}$ and the above process is repeated. This is followed by selection of loads from the range of $0.31-0.4 \mathrm{TL}$ and finally $0.71-0.8 \mathrm{TL}$.

Fourteen scenarios of different transportation requests are tested as given in Error! Reference source not found.. Each transportation request comprises of the location of the origin and destination pair and the load demand relative to the truck capacity (TL). The coordinates for the origins and destinations are randomly and uniformly generated over the range of [-40,40] (miles) for both X and Y coordinates. The depot is located at $[0,0]$ for all scenarios.

Table 1: Fourteen scenarios considered

| Scenario ID | Total number of nodes | Number of origins x Number of <br> destinations |
| :---: | :---: | :---: |
| 1 | 20 | $5 \times 15$ |
| 2 | 30 | $10 \times 20$ |
| 3 | 60 | $20 \times 40$ |
| 4 | 90 | $30 \times 60$ |
| 5 | 100 | $30 \times 70$ |
| 6 | 110 | $40 \times 70$ |
| 7 | 120 | $40 \times 80$ |
| 8 | 130 | $50 \times 80$ |
| 9 | 140 | $50 \times 90$ |
| 10 | 200 | $60 \times 90$ |
| 11 | 300 | $80 \times 120$ |
| 12 | 350 | $110 \times 140$ |
| 13 |  | $120 \times 180$ |
| 14 |  | $150 \times 200$ |

In addition to the PDPSL heuristic, the performance of the M-MSPDP heuristic is also compared with the heuristic provided in Sahin et al. (2013), which uses a Tabu search and Simulated Annealing based (TESA) heuristic.

Comparison with the exact solution method


Figure 2a and b: Absolute performance of the MS-BTS heuristic (solution quality and computational time)
Comparison with other heuristics


Figure 3a and b: Performance of the MS-BTS heuristic in comparison to the PDPSL and TESA heuristic (solution quality and computational time improvement)

From Figure 3, the solution quality goes on decreasing in the order of PDPSL heuristic, TESA heuristic and the MS-BTS heuristic whereas the savings in the computational time increase in the order of the MS-BTS heuristic, TESA heuristic and the PDPSL heuristic. As observed, the quality of the solution obtained by the MS-BTS heuristic decreases with increasing the sample size and it does not vary much beyond the problem size of 250 nodes. This indicates the reliability of the MSBTS heuristic in solving large problems.

## Case Studies

In the evaluation of the Many-to-Many Split Pickup-and-Delivery Problem with Fixed Pairwise Demand (M-MSPD-FPD), we find that by allowing loads to be split at parcel stations and using MS-BTS heuristic there is a small reduction in the number of trucks dispatched (1.25-1.37\%), average VMT (2.16-2.99\%), and total fuel consumption (1.16-3.19\%) for the large customer demands.

In the second case study, we apply the MS-BTS heuristic to the Bike-sharing Rebalancing Problem (BRP) where we use the same problem setting described in Dell'Amico et al. (2016) and generate random datasets to compare our MS-BTS solution. We find that on average the MS-BTS delivers a better solution in slightly lesser time as compared to the Destroy and Repair meta-heuristic proposed by Dell'Amico et al. (2016).

## Conclusion

In this study, we evaluate the performance of the MS-BTS heuristic with numerical experiments in comparison to the exact solution and with PDPSL and the TESA heuristic. We find that the MSBTS heuristic performs well with an acceptable degradation in the solution quality than the exact solution and the PDPSL heuristic, but with an improvement in the computational time. Besides, the MS-BTS heuristic continues to perform well for both small and large problems. Additional evaluation with the TESA heuristic reveals that the MS-BTS heuristic is slightly quicker but with a small degradation in solution quality. We find the MS-BTS to be useful in solving large scale problems for both parcel-delivery and bike rebalancing applications.

This research indicates that the M-MSPDP should be further explored, with opportunities to decide the order of splitting the loads to search the solution space more efficiently. Research may also be conducted in the local improvement techniques order to improve the vehicle routing.

## Acknowledgment

This research was partially supported by the National Science Foundation, award number 1534138, on the Smart CROwdsourced Urban Delivery (CROUD) System.

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